Dirac-Born-Infeld and k-inflation: the CMB anisotropies from string theory

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Abstract. Inflationary models within string theory exhibit unusual scalar field dynamics involving non-minimal kinetic terms and generically referred to as k-inflation. In this situation, the standard slow-roll approach used to determine the behavior of the primordial cosmological perturbations cannot longer be used. We present a generic method, based on the uniform approximation, to analytically derive the primordial power spectra of scalar and tensor perturbations. At leading order, the scalar spectral index, its running and the tensor-to-scalar ratio are modified by the new dynamics. We provide their new expression, correct previous results at next-to-leading order and clarify the definition of what is the tensor-to-scalar ratio when the sound horizon and Hubble radius are not the same. Finally, we discuss the constraints the parameters encoding the non-minimal kinetic terms have to satisfy, such as the sound speed and the energy scale of k-inflation, in view of the fifth year Wilkinson Microwave Anisotropy Probe (WMAP5) data.

1. Introduction

In the context of string theory, cosmic inflation can be achieved through the motion D-branes in higher dimensional warped and compact spacetimes [1]. There, the inflaton appears as the scalar degree of freedom associated with the position of a brane in these extra-dimensions. From a four-dimensional point of view, Lorentz invariance along a D-brane necessarily leads to four-dimensional scalar fields exhibiting non-standard kinetic terms, and more precisely of the Dirac–Born–Infeld (DBI) type [2, 3, 4]. In fact, DBI-inflation belongs to the class of k-inflationary models in which the accelerated expansion of the universe can be driven by the scalar field's kinetic terms instead of its potential energy [5]. Assuming the gravity sector to be described by General Relativity, the action of the "k-inflaton" $\varphi(x^\mu)$ reads

$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} \left[R + 2\kappa P\left(X, \varphi\right) \right], \tag{1}$$

where $\kappa \equiv 8\pi/m_{\mathrm{Pl}}^2$ and $X \equiv -g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi/2$. The quantity $P(X, \varphi)$ is any *acceptable* functional of X and φ (see Ref. [6]). All kinetically modified inflationary models have a "speed of sound"

$$c_{\mathrm{s}}^2 \equiv \frac{P_{,X}}{P_{,X} + 2X P_{,XX}}, \tag{2}$$

which is generically different from the speed of light. In a flat Friedmann-Lemaître–Robertson–Walker (FLRW) universe, it was shown in Ref. [7], that the Mukhanov–Sasaki mode function

$\mu_{\mathrm{S}}(k,\eta) = \zeta\sqrt{2\kappa\epsilon_1}/c_{\mathrm{s}}$ (ζ being the comoving curvature perturbation) satisfies the modified equation of motion

$$\mu_{\mathrm{S}}'' + \left[c_{\mathrm{s}}^2(\eta)k^2 - \frac{\nu^2(\eta) - 1/4}{\eta^2}\right]\mu_{\mathrm{S}} = 0. \tag{3}$$

The effective potential is $(\nu^2 - 1/4)/\eta^2 = [\ln(a\sqrt{\epsilon_1}/c_{\mathrm{s}})]''$, and all derivatives are with respect to conformal time η. The quantity $a(\eta)$ stands for the FLRW scale factor while $\epsilon_1 = -\mathrm{d}\ln H/\mathrm{d}\ln a$ is the first Hubble flow function (H being the Hubble parameter). The standard form of this equation is recovered by setting $c_{\mathrm{s}} = 1$ and can be solved by defining a hierarchy of Hubble flow functions encoding the rate of change of the Hubble parameter and its higher order logarithmic derivatives: $\epsilon_{n+1} \equiv \mathrm{d}\ln|\epsilon_n|/\mathrm{d}\ln a$. Assuming slow-roll, i.e. $\epsilon_n \ll 1$, one can expand the effective potential $(\nu^2 - 1/4)/\eta^2$ and solve order by order Eq. (3) along the lines of Refs. [8, 9, 10]. Generalising this method to the k-inflationary case in which $c_{\mathrm{s}}(\eta)$ is not constant requires some care. Indeed, both the effective potential and the propagation speed are modified. In the following, we use the uniform approximation to solve Eq. (3) order by order to predict the shape of the tensor and scalar primordial fluctuations at the origin of the CMB anisotropies.

2. K-inflationary perturbations

For k-inflation, we can define a new hierarchy encoding the rate of change of the sound speed, the sound flow functions δ_i, defined by

$$\delta_{n+1} = \frac{\mathrm{d}\ln|\delta_n|}{\mathrm{d}\ln a}, \qquad \delta_0 \equiv \frac{c_{\mathrm{sin}}}{c_{\mathrm{s}}}. \tag{4}$$

Expanding both the sound speed and the effective potential in terms of the Hubble and sound flow functions around a particular conformal time η_* gives [11]

$$\nu^2(\eta) = \frac{9}{4} + 3\epsilon_{1*} + \frac{3}{2}\epsilon_{2*} + 3\delta_{1*} + \mathcal{O}(\epsilon\delta) = \nu_*^2 + \mathcal{O}(\epsilon\delta), \quad c_{\mathrm{s}}(\eta) = c_{\mathrm{s}*}\left(1 + \delta_{1*}\ln\frac{\eta}{\eta_*}\right) + \mathcal{O}(\epsilon\delta), \tag{5}$$

where all stars mean that the corresponding function is evaluated at η_*. From these expressions, one can solve Eq. (3) at first order in the flow functions ϵ_i and δ_i by using the uniform approximation [16, 17].

The scalar primordial power spectrum, at first order in Hubble and sound flow functions, then reads [11]

$$\mathcal{P}_\zeta = \frac{H_\diamond^2}{\pi m_{\mathrm{Pl}}^2 \epsilon_{1\diamond} c_{\mathrm{s}\diamond}}\left[1 - 2(D+1)\epsilon_{1\diamond} - D\epsilon_{2\diamond} + (D+2)\delta_{1\diamond} - (2\epsilon_{1\diamond} + \epsilon_{2\diamond} - \delta_{1\diamond})\ln\frac{k}{k_\diamond}\right], \tag{6}$$

where $D = 1/3 - \ln 3$. All diamond indexed quantities are evaluated at the time $\eta_\diamond$ defined to be the time at which a chosen pivot wavenumber $k_\diamond$ crossed the sound horizon during inflation, i.e. the solution of $-k_\diamond\eta_\diamond = 1/c_{\mathrm{s}\diamond}$. The constant factor $18\mathrm{e}^{-3}$ typical of WKB methods has also been absorbed in the definition of $H_\diamond$ [18].

Concerning the tensor modes, their evolution is not affected by the non-standard kinetic terms and their power spectrum remains the same as in the standard case $c_{\mathrm{s}} = 1$. However, an important, and so far overlooked, difference is that the standard tensor power spectrum is evaluated at the time at which the perturbations crossed the Hubble radius during inflation. As a result, it is expressed in terms of the Hubble flow functions evaluated at a different time than $\eta_\diamond$ and one cannot evaluate a tensor-to-scalar ratio by simply dividing both power spectra. Using

the Hubble and sound flow expansion, one can nevertheless express the tensor power spectrum at $\eta_\diamond$. After some algebra, one obtains

$$\mathcal{P}_h(k) = \frac{16H_\diamond^2}{\pi m_{\mathrm{Pl}}^2}\left[1 - 2\left(D + 1 - \ln c_{\mathrm{s}\diamond}\right)\epsilon_{1\diamond} - 2\epsilon_{1\diamond}\ln\frac{k}{k_\diamond}\right]. \tag{7}$$

We immediately see that the speed of sound influences $\mathcal{P}_h$, and this effect becomes all the more so important than c_{s} is small. The above expression explains the numerical results on the tensor-to-scalar ratio discussed in Ref. [19].

From Eqs. (6) and (7), one can deduce the scalar spectral index n_{s}, its running α_{s} and the tensor-to-scalar ratio at next-to-leading order [10, 11]

$$\begin{aligned}
n_{\mathrm{s}} - 1 &\equiv \left(\frac{\mathrm{d}\ln\mathcal{P}_\zeta}{\mathrm{d}\ln k}\right)_{k=k_\diamond} = -2\epsilon_{1\diamond} - \epsilon_{2\diamond} + \delta_{1\diamond} - 2\epsilon_{1\diamond}^2 - (2D+3)\epsilon_{1\diamond}\epsilon_{2\diamond} + 3\epsilon_{1\diamond}\delta_{1\diamond} + \epsilon_{2\diamond}\delta_{1\diamond} \\
&\qquad\qquad - D\epsilon_{2\diamond}\epsilon_{3\diamond} - \delta_{1\diamond}^2 + (D+2)\delta_{1\diamond}\delta_{2\diamond}, \\
\alpha_{\mathrm{s}} &\equiv \left(\frac{\mathrm{d}^2\ln\mathcal{P}_\zeta}{\mathrm{d}\ln^2 k}\right)_{k=k_\diamond} = -2\epsilon_{1\diamond}\epsilon_{2\diamond} - \epsilon_{2\diamond}\epsilon_{3\diamond} + \delta_{1\diamond}\delta_{2\diamond}, \\
r &\equiv \left.\frac{\mathcal{P}_h}{\mathcal{P}_\zeta}\right|_{k=k_\diamond} = 16c_{\mathrm{s}\diamond}\epsilon_{1\diamond}\left[1 + 2\epsilon_{1\diamond}\ln c_{\mathrm{s}\diamond} + D\epsilon_{2\diamond} - (D+2)\delta_{1\diamond}\right].
\end{aligned} \tag{8}$$

The spectral index and running correct previous results at next-to-leading order [12, 13, 14], which were assuming c_{s} constant, and match with another method proposed by Kinney and Tzirakis in Ref. [15]. The term in $\ln c_{\mathrm{s}\diamond}$ in the tensor-to-scalar ratio has to be considered as soon as $c_{\mathrm{s}\diamond}$ becomes small enough.

3. Conclusion

From the scalar and tensor power spectra given above, one can compare the predicted CMB anisotropies with the current data. In Ref. [20], we have performed a Monte–Carlo–Markov–Chains analysis of the WMAP5 data [21] against the k-inflationary power spectra. At 95% of confidence, the flow parameters and the energy scale of k-inflation have to verify

$$0.003 \leq 2(\epsilon_{1\diamond} - \delta_{1\diamond}) + (\epsilon_{2\diamond} + \delta_{1\diamond}) \leq 0.075, \quad \log(\epsilon_{1\diamond}c_{\mathrm{s}\diamond}) \leq -2.3, \quad \ln\left(10^5\frac{H_\diamond}{m_{\mathrm{Pl}}}\right) \leq -0.59. \tag{9}$$

Notably, due to the new degree of freedom introduced by c_{s}, we do not longer find any bound on $\epsilon_{1\diamond}$ alone. The class of k-inflationary models is thus weakly constrained by CMB data. Let us however mention that the subclass of DBI models generate a large amount of non-Gaussianity when c_{s} becomes small [22]. In this later case, one can show that the current WMAP5 bounds on non-Gaussianity imposes that $\log\epsilon_{1\diamond} \leq -1.1$, at two-sigma [20].

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